

Enhancing quantum phase slips via multiple nanowire junctions

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Realization of robust coherent quantum phase slips represents a significant experimental challenge. Here we propose a novel structure that utilizes multiple nanowire junctions to enhance the process of quantum phase slips in a phase-slip flux qubit. In this structure, each superconducting island separated by two adjoining junctions is biased by a gate voltage, which allows us to have a tunable phase-slip rate. Furthermore, we describe how to couple two phase-slip flux qubits via the mutual inductance between them. These two inductively coupled phase-slip flux qubits are dual to two capacitively coupled charge qubits.

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I. INTRODUCTION

In a superconducting wire, the requirement of current conservation leads to a constraint, $\psi_0^2 \nabla \phi = \text{constant}$, relating the amplitude ψ_0 and the phase ϕ of the superconducting order parameter $\psi = \psi_0 e^{i\phi}$. When the amplitude goes to zero, the phase is allowed to have rapid changes (i.e., the slips) by $\pm 2\pi$. This phenomenon is known as the phase slip [1–3]. For a long time, achieving coherent quantum phase slips has been an interesting but challenging topic. Traditional methods [4] rely on the detection of phase-slip changes in resistance measurements of superconducting wires. However, one cannot fully reveal the quantum nature of the phase-slip process [5, 6] because phase slips can also be activated by thermodynamic fluctuations that contribute to the residual resistance of superconducting wires [7–9]. A more sophisticated method for detecting coherent quantum phase slips is to engineer a device known as a quantum phase-slip junction [10]. This phase-slip junction can play the role of a Josephson junction [11] in a superconducting flux qubit to form a new kind of qubit known as the phase-slip flux qubit [10, 12, 13]. Compared with conventional superconducting qubits for quantum information processing [14–16], this phase-slip qubit is insensitive to the charge noise. Moreover, similar to Josephson junctions for accurate standards of voltage [17], the flux-charge duality [12, 18] may make this phase-slip qubit a promising device for providing quantum current standard [19]. In recent experiments, highly disordered indium oxide (InO_x) and niobium nitride (NbN) nanowires were utilized to achieve the phase-slip flux qubit [20, 21]. In addition, a single-charge transistor based on quantum phase slips was realized [22], which is dual to dc SQUID and can be operated as an electrometer. These experiments represent an important step towards the ap-

plications of phase-slip circuits in quantum information processing and quantum metrology.

Physically, achieving robust coherent quantum phase slips is still an experimental challenge. The major difficulty comes from quasiparticle dissipations in nanowires or vortex cores which make the phase-slip rate imperceptible. These dissipations can be suppressed in a highly disordered superconductor near the superconductor-insulator transition, where electrons are localized and quantum fluctuations of the order parameter are prominent. However, quantum fluctuations are still weak in disordered bulk superconductors, but they can become significantly strong in some disordered nanowires where the localization length is comparable to the coherence length. It was shown that the phase-slip rate can be increased by raising the disorders in the superconducting nanowires [23, 24], but this is limited since too many disorders in the nanowires can yield strong Coulomb interactions and then destroy the superconductivity [25]. Alternatively, the phase-slip rate can be increased by using a longer nanowire [5, 10], but this is also limited because the quantum fluctuations needed for the emergence of the phase slips become weakened and even disappeared when increasing the length of the nanowire (e.g., the MoGe nanowire can be as long as $L \sim 200$ nm [5, 26] for keeping the needed quantum fluctuations). Therefore, an alternative method to enhance the phase-slip rate is strongly desired.

In this paper, we propose to use multiple nanowire junctions to enhance the quantum phase slips in a phase-slip flux qubit. This is particularly important when a single junction exhibits only weak phase slips, because such multiple junctions can collectively give rise to a required large phase-slip rate to demonstrate appreciable quantum phase slips. In this phase-slip qubit, each superconducting island separated by two adjoining phase-

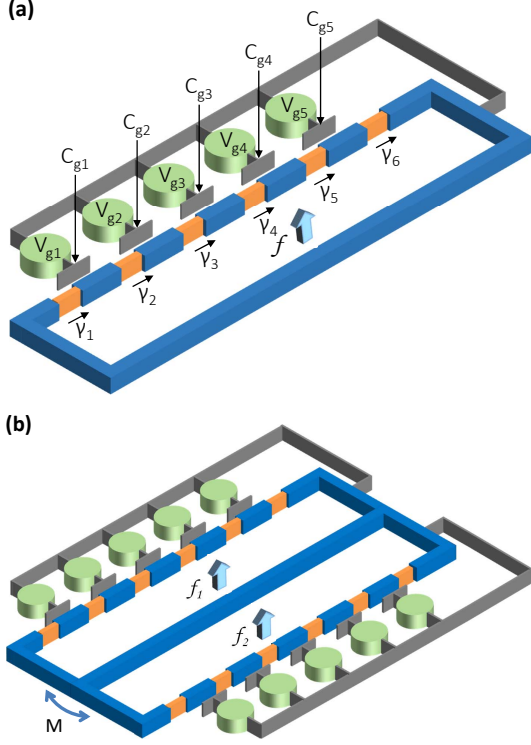


FIG. 1. (a) Schematic diagram of a multi-junction phase-slip flux qubit, where γ_i , for $i = 1, 2, \dots, m$, represents the phase drop across the i th junction in the loop, and $f \equiv \Phi_{\text{ext}}/\Phi_0$ is the reduced magnetic flux applied to the loop. Each superconducting island between two adjoining phase-slip junctions is controlled by the gate voltage V_{gl} via a gate capacitance C_{gl} , where $l = 1, 2, \dots, m-1$. (b) Two multi-junction phase-slip flux qubits coupled by a mutual inductance M between them.

slip junctions is also biased by a gate voltage, so as to achieve a tunable phase-slip rate. Moreover, we propose to couple two multi-junction phase-slip flux qubits via the mutual inductance between them. These two inductively coupled phase-slip flux qubits are dual to two charge qubits coupled via a mutual capacitance. Our proposed multi-junction setup has distinct advantages over a single phase-slip junction, because it can achieve a large effective phase-slip rate to demonstrate appreciable phase slips and make more materials (including those with weak phase slips) usable in designing superconducting quantum circuits.

II. MULTI-JUNCTION PHASE-SLIP FLUX QUBIT

A. Hamiltonian

The proposed multi-junction phase-slip flux qubit is schematically shown in Fig. 1(a), where a superconducting loop is interrupted by m phase-slip junctions. The

voltage drop across each phase-slip junction is given by [12] $V_i = V_{Ci} \sin(2\pi q_i)$ for $i = 1, 2, \dots, m$, where $V_{Ci} = 2\pi E_{Si}/2e$, with E_{Si} being the phase-slip rate, is the critical voltage of the i th junction and q_i is the number of Cooper pairs tunneling through the i th phase-slip junction. Every two adjoining phase-slip junctions are connected by a superconducting island biased by a gate voltage V_{gl} via a gate capacitance C_{gl} ($l = 1, 2, \dots, m-1$). For simplicity, we consider the case with identical phase-slip junctions (i.e., $E_{Si} = E_S$), and equal gate capacitances and biasing voltages (i.e., $C_{gl} = C_g$ and $V_{gl} = V_g$). The reduced offset charge on each island is $N_g \equiv C_g V_g/2e$ and the supercurrent through each junction is $I = 2e\dot{q}_i$. The phase drop across the i th phase-slip junction with the kinetic inductance L_{ki} is $\gamma_i = 2\pi (L_{ki}I/\Phi_0)$, and the phase drop related to the geometry inductance L_g of the loop is $\gamma_g = 2\pi (L_g I/\Phi_0)$, where $\Phi_0 \equiv h/2e$ is the flux quanta.

In the fluxoid (flux quanta) representation, the Hamiltonian of the multi-junction phase-slip flux qubit has the same form as the single-junction phase-slip flux qubit [10, 12]:

$$H_q = E_L (n - f)^2 - \frac{1}{2} E_{\text{eff}} \times \sum_n (|n+1\rangle \langle n| + |n\rangle \langle n+1|), \quad (1)$$

but the inductive energy is different and the phase-slip rate is replaced by an effective one (see Appendix A):

$$E_L = \frac{\Phi_0^2}{2(L_g + \sum_i L_{ki})}, \quad (2)$$

$$E_{\text{eff}}(N_g) = E_S \frac{\sin(m\pi N_g)}{\sin(\pi N_g)}, \quad (3)$$

where the fluxoid states $\{|n\rangle\}$ are the eigenstates of $n \equiv \Phi/\Phi_0$, and $f \equiv \Phi_{\text{ext}}/\Phi_0$ is the reduced externally-applied flux of the loop. Compared with the single-junction phase-slip flux qubit [10], E_L can be decreased by using multiple phase-slip junctions, so that the effect of the flux noise on the qubit is suppressed. Also, note that the effective phase-slip rate depends on the reduced offset charge $N_g = C_g V_g/2e$. Therefore, in comparison with the single-junction phase-slip flux qubit [10, 12], the proposed multi-junction phase-slip flux qubit can be tuned by both f and N_g , i.e., the externally applied magnetic flux Φ_{ext} in the loop and the gate voltage V_g biased on each island.

In Fig. 2, the effective phase-slip rate in Eq. (3) is shown for different numbers of phase-slip junctions. In the single-junction case of $m = 1$, E_{eff} is reduced to the constant E_S , but it becomes N_g -dependent when $m \geq 2$. In particular, when tuning the reduced offset charge N_g to be equal to an integer number $k \in \mathbb{N}$, E_{eff} reaches its maximum value

$$\lim_{N_g \rightarrow k \in \mathbb{N}} E_{\text{eff}}(N_g) = m E_S, \quad (4)$$

i.e., the phase-slip rate is magnified m times.

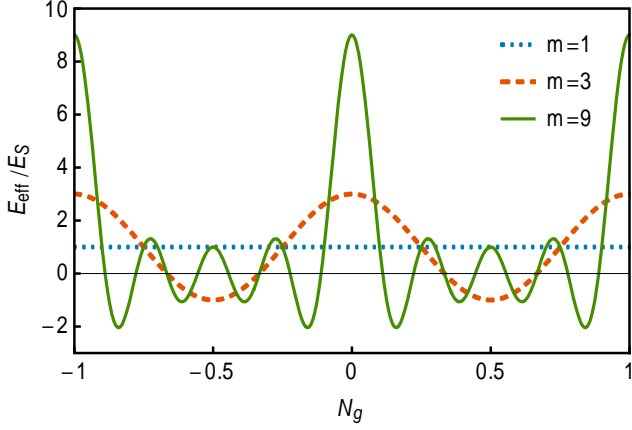


FIG. 2. Effective phase-slip rate E_{eff} of a multi-junction phase-slip flux qubit (in units of E_S) versus the reduced offset charge N_g ($\equiv C_g V_g / 2e$).

B. Advantages of using multiple phase-slip junctions

In addition to the suppression of the flux-noise effect on the qubit, the most important advantage of using multiple phase-slip junctions is the significant increase of the phase-slip rate. To implement a phase-slip flux qubit, the phase-slip rate should have a large value to achieve appreciable quantum tunneling between fluxoid states $|n\rangle$ and $|n+1\rangle$.

For a single junction, the phase-slip rate E_S used to characterize the coherent quantum phase slips in a phase-slip flux qubit has the following form [5, 10]:

$$E_S = 1.5c \frac{k_B T_c L}{\hbar \xi} \sqrt{N_\xi} e^{-0.3dN_\xi}, \quad (5)$$

where L and ξ are the physical length and the coherence length of the superconducting wire, respectively, T_c is the critical temperature, c and d are two constants of order unity [10], and $N_\xi \equiv R_q / R_\xi$ is the number of effective conductive channels (or dimensionless conductance), which is defined by the ratio between the resistance quantum $R_q \equiv h/4e^2$ and the resistance R_ξ of a superconducting wire with the length equal to ξ . In order to increase E_S , one can raise the factor $e^{-0.3dN_\xi} \sqrt{N_\xi} / \xi$. This requires using an disordered material (i.e., small N_ξ) for the junction. However, the increase of the phase-slip rate by raising the disorders in the junction is limited because too many disorders in the material can lead to strong Coulomb interactions and thus destroy the superconductivity [25]. Alternatively, one can increase the phase-slip rate E_S by using a longer junction [see Eq. (5)]. However, this is also limited because the quantum fluctuations needed for the emergence of the phase slips become weakened and even disappeared as the junction becomes significantly longer. For example, the length of the junction can only be as long as $L \sim 200$ nm for a MoGe nanowire [5, 26].

From Eq. (4), instead of using a single junction, we can use multiple phase-slip junctions to achieve a large effective phase-slip rate E_{eff} , although each junction may have a small phase-slip rate E_S but both the disorders and the length are within the allowed range for the phase slips to occur. Indeed, a single junction may not exhibit appreciable phase slips when its phase-slip rate is small, but our result reveals that multiple junctions can collectively do. Therefore, the multiple-junction setup can not only achieve a large effective phase-slip rate to demonstrate appreciable phase slips, but also make the materials with weak phase slips usable in superconducting quantum circuits.

III. TWO INDUCTIVELY COUPLED MULTI-JUNCTION PHASE-SLIP FLUX QUBITS

In order to implement a nontrivial two-qubit quantum gate, one needs a pair of coupled phase-slip flux qubits. Here, as shown in Fig. 1(b), we consider two multi-junction phase-slip flux qubits coupled via a mutual inductance between them.

The phase drop across the i th phase-slip junction in the left (right) superconducting loop in Fig. 1(b) is

$$\gamma_{i,1(2)} = 2\pi \frac{L_{ki,1(2)} I_{1(2)}}{\Phi_0}, \quad (6)$$

where $L_{ki,1(2)}$ is the kinetic inductance of the i th junction in the left (right) phase-slip flux qubit, and $I_{1(2)}$ is the corresponding supercurrent. The phase drop related to the geometry inductance of each loop is

$$\gamma_{g,1(2)} = 2\pi \frac{L_{g,1(2)} I_{1(2)}}{\Phi_0}. \quad (7)$$

Also, the magnetic flux in each loop is affected by an adjacent loop through the mutual inductance M between them. The resulting phase drops are given by

$$\gamma_{12} = 2\pi \frac{M I_2}{\Phi_0}, \quad \gamma_{21} = 2\pi \frac{M I_1}{\Phi_0}. \quad (8)$$

Now the fluxoid quantization condition for each superconducting loop becomes

$$2\pi f_{1(2)} - \sum_i \gamma_{i,1(2)} - \gamma_{g,1(2)} + \gamma_{12(21)} = 2\pi n_{1(2)}. \quad (9)$$

Here the eigenvalues of $n_{1(2)} \equiv \Phi_{1(2)} / \Phi_0$ are integers, and $f_{1(2)} \equiv \Phi_{1(2),\text{ext}} / \Phi_0$ is the reduced magnetic flux, with $\Phi_{1(2),\text{ext}}$ being the externally applied flux in the left (right) loop.

From Eqs. (6)-(8), we have

$$L_1 I_1 - M I_2 = \frac{\Phi_0}{2\pi} \left(\sum_i \gamma_{i,1} + \gamma_{g,1} - \gamma_{12} \right), \quad (10)$$

$$L_2 I_2 - M I_1 = \frac{\Phi_0}{2\pi} \left(\sum_i \gamma_{i,2} + \gamma_{g,2} - \gamma_{21} \right), \quad (11)$$

where $L_{1(2)} = \sum_i L_{ki,1(2)} + L_{g,1(2)}$ is the total inductance of the left (right) loop. Using the fluxoid quantization condition in Eq. (9), we can solve Eqs. (10) and (11) as

$$I_1 = \frac{L_2 (f_1 - n_1) + M (f_2 - n_2)}{\Lambda_-} \Phi_0, \quad (12)$$

$$I_2 = \frac{L_1 (f_2 - n_2) + M (f_1 - n_1)}{\Lambda_-} \Phi_0, \quad (13)$$

with $\Lambda_- = L_1 L_2 - M^2$.

Similar to the single phase-slip flux qubit (see Appendix A), the Hamiltonian of the two inductively coupled phase-slip flux qubits is given by

$$H_{q-q} = \sum_{j=1}^2 \left[\frac{1}{2} L_j I_j^2 - E_{\text{eff},j} \cos(2\pi Q_j) \right] + M I_1 I_2, \quad (14)$$

where

$$E_{\text{eff},j} = E_{S,j} \frac{\sin(m_j \pi N_{g,j})}{\sin(\pi N_{g,j})}, \quad (15)$$

with $N_{g,j} = C_{g,j} V_{g,j} / 2e$ being the reduced offset charge on each island in the j th multi-junction phase-slip flux qubit, and m_j the number of phase-slip junctions in the qubit; $Q_{1(2)} = \sum_i q_{i,1(2)} / m_{1(2)}$, with $q_{i,1(2)}$ being the number of Cooper pairs tunneling through the i th phase-slip junction in the left (right) qubit. In the fluxoid representation, the Hamiltonian (14) can be written as

$$\begin{aligned} H_{q-q} = & \sum_{j=1}^2 E_{L,j} (n_j - f_j)^2 + E_{12} (n_1 - f_1) (n_2 - f_2) \\ & - \frac{1}{2} \sum_{j=1}^2 \sum_{n_j} E_{\text{eff},j} (|n_j + 1\rangle \langle n_j| + |n_j\rangle \langle n_j + 1|), \end{aligned} \quad (16)$$

where

$$E_{L,1(2)} = \frac{(2\Lambda_+ - \Lambda_-) L_{2(1)}}{2\Lambda_-^2} \Phi_0^2, \quad (17)$$

$$E_{12} = \frac{2\Lambda_+ + \Lambda_-}{\Lambda_-^2} \sqrt{\frac{\Lambda_+ - \Lambda_-}{2}} \Phi_0^2, \quad (18)$$

with $\Lambda_+ = L_1 L_2 + M_{12}^2$. These two inductively coupled phase-slip flux qubits are dual to two capacitively coupled charge qubits [27, 28].

For a multi-junction phase-slip flux qubit described by Hamiltonian (1), if it is in the flux regime with $E_L \gg E_{\text{eff}}$, the two fluxoid states $|0\rangle$ and $|1\rangle$ are important when f is tuned to be around the optimal point

$f \sim \frac{1}{2}$. The Hamiltonian (1) can be reduced to [10] $H = E_L (f - \frac{1}{2}) \sigma_z - \frac{1}{2} E_{\text{eff}} \sigma_x$, where $\sigma_z = |0\rangle \langle 0| - |1\rangle \langle 1|$, and $\sigma_x = |0\rangle \langle 0| + |1\rangle \langle 1|$. For the two inductively coupled phase-slip flux qubits, let us also consider the flux regime with $E_{L,j} \gg E_{\text{eff},j}, E_{12}$. Around the optimal point $f_j \sim \frac{1}{2}$ for each qubit, the Hamiltonian (16) is reduced to

$$\begin{aligned} H_{q-q} = & \left[E_{L,1} \left(f_1 - \frac{1}{2} \right) - \frac{1}{2} E_{12} \left(f_2 - \frac{1}{2} \right) \right] \sigma_z^{(1)} \\ & + \left[E_{L,2} \left(f_2 - \frac{1}{2} \right) - \frac{1}{2} E_{12} \left(f_1 - \frac{1}{2} \right) \right] \sigma_z^{(2)} \\ & - \frac{1}{2} \sum_{j=1}^2 E_{\text{eff},j} \sigma_x^{(j)} + \frac{1}{4} E_{12} \sigma_z^{(1)} \sigma_z^{(2)}. \end{aligned} \quad (19)$$

From Eq. (19), it can be seen that the mutual inductance yields a ZZ-type interaction between the two phase-slip flux qubits. Also, it shifts the energy level of each qubit.

IV. CONCLUSION

To conclude, we have described a novel structure for enhancing the quantum phase slips in a phase-slip flux qubit by using multiple junctions of nanowires. Our results show that the collective effect of the multiple junctions gives rise to a large phase-slip rate that can lead to appreciable number of quantum phase slips events. The effective phase-slip rate can be adjusted via the gate voltage on each island between a pair of adjoining phase-slip junctions. Consequently, the phase-slip flux qubit can be controlled by this gate voltage, apart from the magnetic flux applied to the qubit loop. Furthermore, we have proposed to couple two multi-junction phase-slip flux qubits via the mutual inductance between them, which are dual to two capacitively coupled charge qubits. Currently, many materials can only exhibit weak signals of quantum phase slips, which makes them not ideal for quantum information processing. Our proposed multi-junction structure not only provides a large effective phase-slip rate that can enhance appreciable signals of quantum phase slips, but also potentially allows these materials become applicable as robust elements in superconducting circuits.

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Appendix A: Derivation of the Hamiltonian for a multi-junction phase-slip flux qubit

For the multi-junction phase-slip flux qubit, the kinetic energy can be written as

$$T = \frac{1}{2} \left(L_g + \sum_{i=1}^m L_{ki} \right) I^2, \quad (\text{A1})$$

and the potential energy is

$$U = \sum_{i=1}^m \int IV_i dt = \sum_{i=1}^m E_{Si} [1 - \cos(2\pi q_i)]. \quad (\text{A2})$$

Here we consider the simple case with identical phase-slip junctions (i.e., $E_{Si} = E_S$, with $i = 1, 2, \dots, m$) and equal gate capacitances and gate voltages (i.e., $C_{gl} = C_g$ and $V_{gl} = V_g$, with $l = 1, 2, \dots, m-1$). Under a constant voltage for each gate, the charge imbalance implies the following relation:

$$q_i - q_{i-1} = N_g, \quad (\text{A3})$$

where $N_g = C_g V_g / 2e$. The terms in the total potential energy U in Eq. (A2) can be summed to yield an analytic form after neglecting some constant terms:

$$\begin{aligned} U &= -\frac{E_S}{2 \sin(2\pi N_g)} \sum_{i=1}^m \{ \sin[2\pi(q_i + N_g)] \\ &\quad - \sin[2\pi(q_i - N_g)] \} \\ &= -\frac{E_S}{2 \sin(2\pi N_g)} \{ \sin(2\pi q_m) - \sin(2\pi q_1) \\ &\quad + \sin[2\pi(q_m + N_g)] - \sin[2\pi(q_1 - N_g)] \} \\ &= -\frac{E_S \cos(2\pi Q)}{\sin(2\pi N_g)} \{ \sin[(m-1)\pi N_g] \\ &\quad + \sin[(m+1)\pi N_g] \} \\ &= -E_{\text{eff}} \cos(2\pi Q), \end{aligned} \quad (\text{A4})$$

where

$$E_{\text{eff}}(N_g) = E_S \frac{\sin(m\pi N_g)}{\sin(\pi N_g)}, \quad (\text{A5})$$

and Q is the average number of Cooper pairs defined by

$$Q \equiv \frac{1}{m} \sum_{i=1}^m q_i. \quad (\text{A6})$$

Thus, the Lagrangian of the multi-junction phase-slip flux qubit is given by

$$\begin{aligned} L &= T - U \\ &= \frac{1}{2} \left(L_g + \sum_{i=1}^m L_{ki} \right) I^2 + E_{\text{eff}} \cos(2\pi Q), \end{aligned} \quad (\text{A7})$$

where $I = 2e\dot{q}_i = 2e\dot{Q}$, because $\dot{q}_i = \dot{q}_{i+1}$.

When choosing Q as the canonical coordinate, the corresponding canonical momentum is

$$P = \frac{\partial L}{\partial \dot{Q}} = \left(L_g + \sum_{i=1}^m L_{ki} \right) 2eI. \quad (\text{A8})$$

The fluxoid quantization condition is

$$2\pi f - \sum_{i=1}^m \gamma_i - \gamma_g = 2\pi n, \quad (\text{A9})$$

where $\gamma_i = 2\pi(L_{ki}I/\Phi_0)$, and $\gamma_g = 2\pi(L_g I/\Phi_0)$. It follows that the supercurrent can be expressed as

$$I = \frac{\Phi_0}{L_g + \sum_{i=1}^m L_{ki}} (f - n). \quad (\text{A10})$$

Therefore, the Hamiltonian of the multi-junction phase-slip flux qubit is obtained as

$$H = P\dot{Q} - L = E_L (n - f)^2 - E_{\text{eff}} \cos(2\pi Q), \quad (\text{A11})$$

with E_L given by Eq. (2). This Hamiltonian is just Eq. (1) in the fluxoid representation.

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